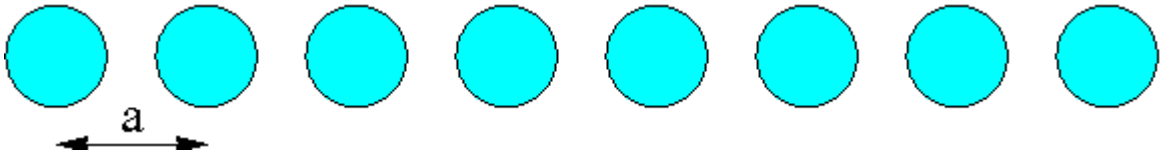
Translational Invariance: First Brillouin Zone



$$\rho(x+ma) = \sum_n \rho_n e^{iG_n(x+ma)} = \sum_n \rho_n e^{iG_n(x)} e^{iG_n ma}$$

$$= \sum_n \rho_n e^{iG_n(x)} = \rho(x)\,,$$

From the periodicity of the lattice it follows that $e^{iG_n ma} = 1$ or $G_n = 2n\pi/a$ where *n* is an integer.

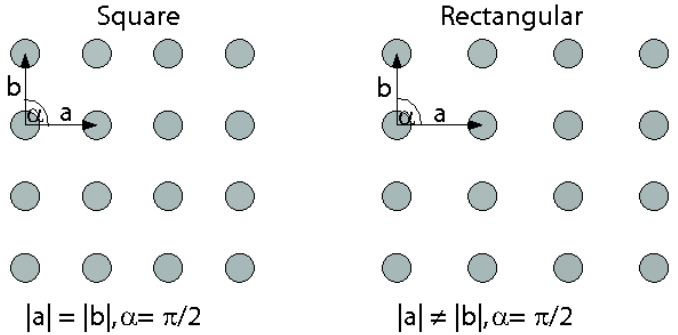
$$\mathsf{G} \cdot \mathsf{r}_n = 2\pi m \quad m \in \mathcal{Z}$$

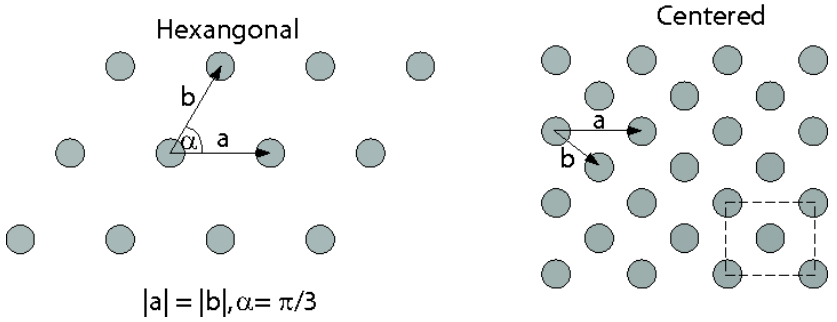
$$\mathbf{G} = h\mathbf{g}_1 + \mathbf{g}_2 + l\mathbf{g}_3$$

The condition of periodicity then requires that

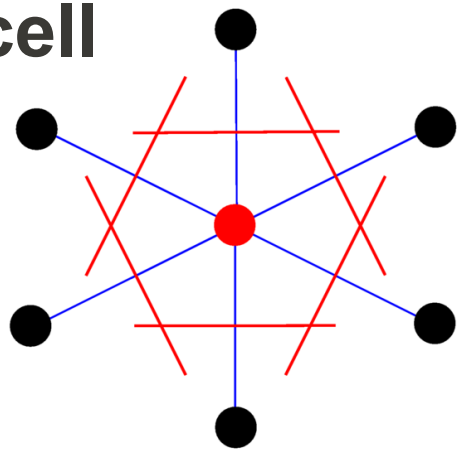
$$(h\mathbf{g}_1 + k\mathbf{g}_2 + l\mathbf{g}_3) \cdot n_1\mathbf{a}_1 = 2\pi m \quad m \in \mathcal{Z}$$

$$\mathbf{g}_1 = 2\pi \frac{\mathbf{a}_2 \times \mathbf{a}_3}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)} \quad \text{plus cyclic permutations}$$

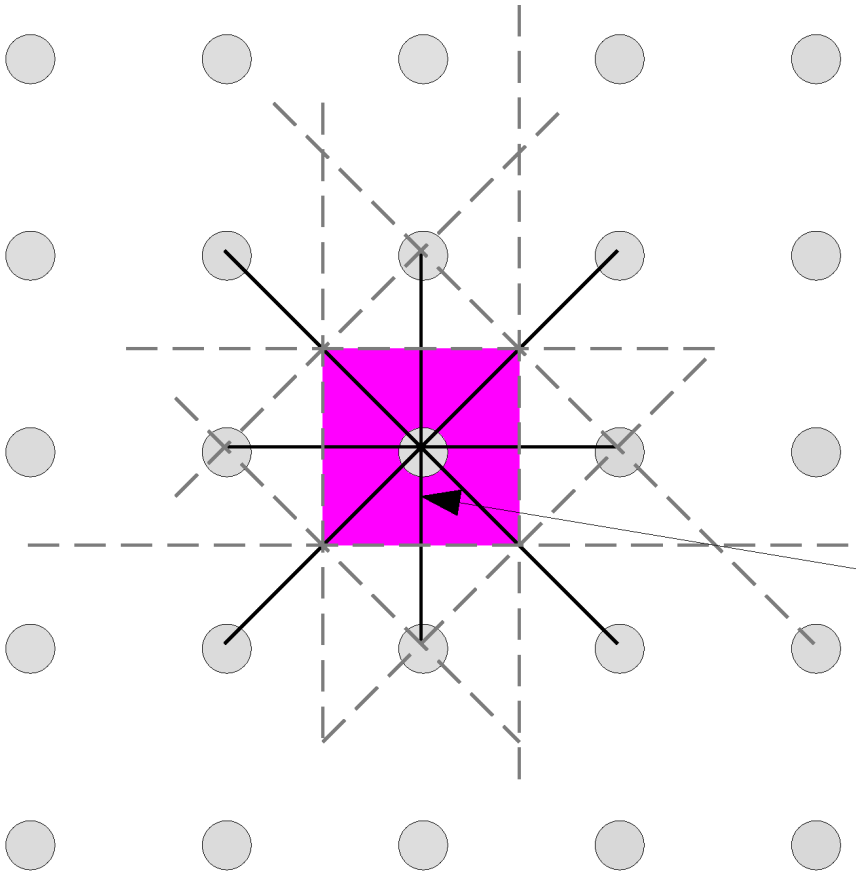




Wigner-Seitz cell



First Brillouin Zone

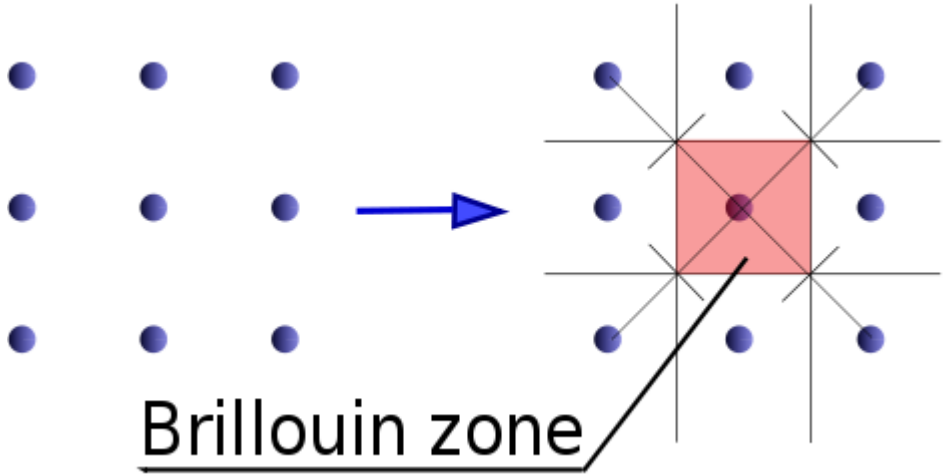


G vector

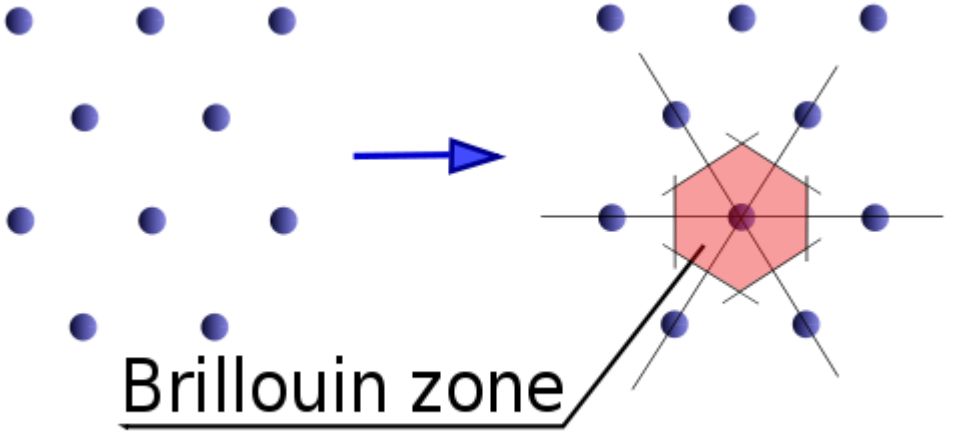
Bisector

First Brillouin Zone

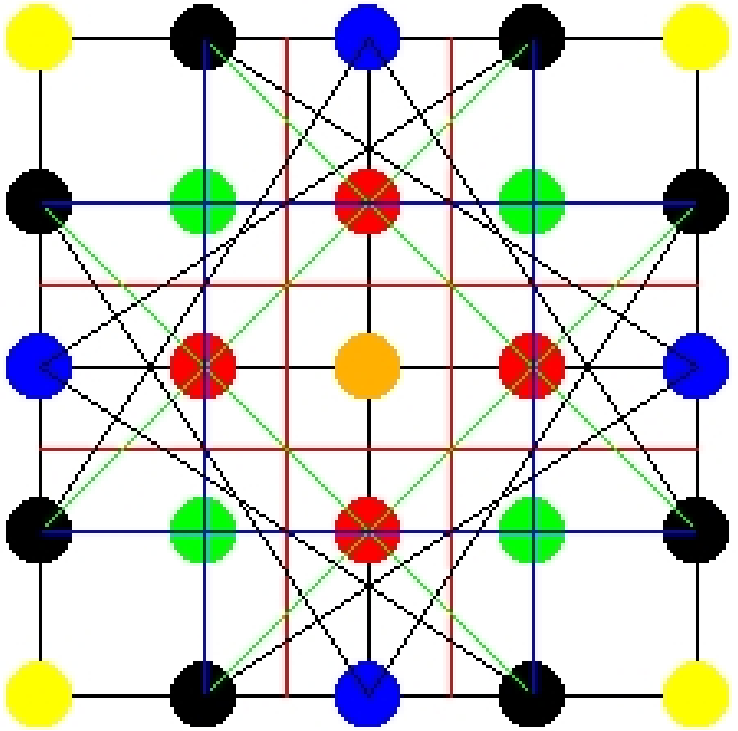
Square Lattice



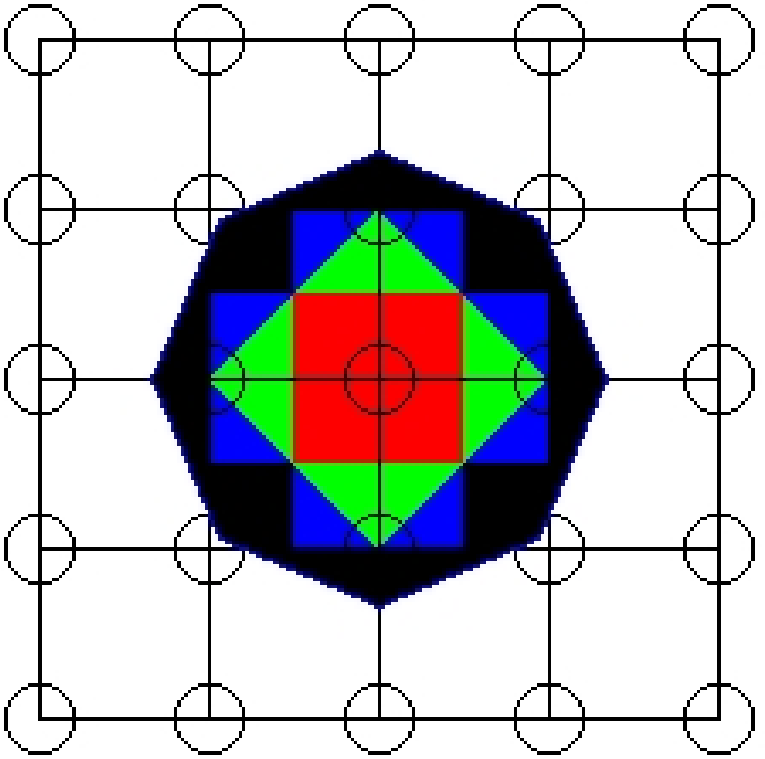
Hexagonal Lattice



Higher Brillouin Zones



The Nearest through Fifth Nearest Neighbors
for a Point in a Square Lattice
and Their Bragg Lines



First Four Brillouin Zones for a Square Lattice